

GRAPH THEORY

NODAL DEGREE:	the number of edges connected to a node
NODAL DEGREE SET:	a set of all nodal degrees
ISOLATED NODE:	a node that has no edges attached to it
PARALLEL EDGES:	edges that connect the same two nodes
LOOP:	an edge connecting a node with itself (counts as 2 degrees)
SIMPLE GRAPH:	a graph that has no parallel connections or loops
NON-SIMPLE GRAPH:	a graph that has at least one parallel connection or loop
PLANAR GRAPH:	a graph with no intersecting edges
NON-PLANAR GRAPH:	a graph that can be drawn with no intersecting edges in 3 dimensions
DIAGRAPH:	a graph that has directions
CONNECTED GRAPH:	a graph where there is path between any two nodes
COMPLETE GRAPH:	a simple graph where each node has a maximum number of edges
PATH:	no repeated edges
EULER PATH:	no repeated edges; <u>all edges</u> are used
SIMPLE PATH:	no repeated edges or nodes; (except when 1 st node = last node)
HAMILTON PATH:	no repeated edges or nodes; <u>all nodes</u> are used,
CIRCUIT:	no repeated edges; 1 st node = last node
EULER CIRCUIT:	no repeated edges; <u>all edges</u> are used; 1 st node = last node
SIMPLE CIRCUIT:	no repeated edges or nodes; 1 st node = last node
HAMILTON CIRCUIT:	no repeated edges or nodes; <u>all nodes</u> are used; 1 st node = last node

EULER PATH exists if and only if:

1. the graph is **connected**
2. the degrees of all nodes are **even**; or only 2 nodes have odd degrees

When looking for a EULER PATH:

If all nodes have even degrees, start at any node.

If 2 nodes have odd degrees, start with either one of the nodes that has an odd degree.

EULER CIRCUIT exists if and only if:

1. the graph is **connected**
2. the degrees of all nodes are **even**

HAMILTON CIRCUIT exists if and only if:

1. every node is adjacent to at least half the nodes in the graph (Dirac's Theorem)

Two graphs are EQUIVALENT if and only if:

1. both graphs are **connected** or **disconnected**;
2. both graphs have an **equal** set of nodal degrees (! order does matter !)

Observations about ADJACENCY MATRICES:

1. if the matrix is **symmetric**, it is a graph with **no directions**
2. if all entries are no bigger than 1, it is a **simple** graph

Observation about INCIDENCE MATRICES:

1. if the matrix has negative entries, it is a graph **with directions**

IMPORTANT FORMULA: number of edges = nodal degrees / 2